# Best multiple criteria choice: the Rubis outranking method

## MICS: AlgorithmicDecision Theory

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April 14, 2013





# Introduction







# The outranking situation

## Definition

- We say that "a decision alternative *a* outranks a decision alternative *b*" if and only:
  1. There is a weighted majority of criteria (or objectives) who warrant that *a* is perceived at least as good as *b* and,
  2. No considerable negative performance difference is observed between *a* and *b* on any criterion (or objective).
- We say that "a decision alternative *a* does not outrank a decision alternative *b*" if and only if:
  3. There is only a weighted minority of criteria (or objectives) who warrant that *a* is perceived at least as good as *b* and,
  4. No considerable positive performance difference between *a* and *b* is observed on any criterion (or objective).
- Cases (2), respectively (4), are called veto, respectively counter–veto situations.



# Best office choice

- Let us reconsider the best office choice problem from lecture 5.
- Below the performances of the seven potential office sites with respect to the three objectives:

| Site | Costs (in €) | Turnover (0-81%) | Work Cond. (0-19%) |
|---|---|---|---|
| A | -35 000 | 70.6 | 10.2 |
| B | -17 800 | 29.5 | 9.9 |
| C | -6 700 | 43.8 | 3.6 |
| D | -14 100 | 42.3 | 10.0 |
| E | -34.800 | 49.1 | 15.7 |
| F | -18 600 | 16.1 | 4.8 |
| G | -12 000 | 49.1 | 10.4 |

## Significant preferential judgment

### Example

- The CEO of the SME judges the "*Costs*" and the cumulated "*Benefits*" objectives ("*Turnover*" and "*Working Conditions*") to be equi-significant for selecting the best office site.
- Hence, he considers that a concordant preferential judgment with respect to "*Costs*" and one of the two "*Benefits*" objectives is significant for him.





## Certainly confirmed outranking situation

### Example

| Site | Costs (in €) | Turnover (0-81) | Work Cond. (0-19) |
|---|---|---|---|
| G | -12 000 | 49.1 | 10.4 |
| F | -18 600 | 16.1 | 4.8 |

- Site $G$ certainly outranks site $F$ as $G$ is at least as well performing than $F$ on all three objectives (unanimous concordance = Pareto dominance).





## Positively confirmed outranking situation

### Example

| Site | Costs (in €) | Turnover (0-81) | Work Cond. (0-19) |
|---|---|---|---|
| C | -6 700 | 43.8 | 3.6 |
| B | -17 800 | 29.5 | 9.9 |

- Site $C$ outranks site $B$ as $C$ is at least as well performing than $B$ on objective "*Costs*" (-6 700 against -17 800) and on objective "*Turnover*" (43.8 against 29.5).





## Confirmed outranking situation

### Example

| Site | Costs (in €) | Turnover (0-81) | Work Cond. (0-19) |
|---|---|---|---|
| G | -12 000 | 49.1 | 10.4 |
| A | -35 000 | 70.6 | 10.2 |

- Site $G$ outranks site $A$, as $G$ is at least as well performing than $A$ on objective "*Costs*" (-12 000 against -35 000) and objective "*Work Cond.*" (10.4 against 10.2).

# Positively rejected outranking situation

## Example

| Site | Costs (in €) | Turnover (0-81) | Work Cond. (0-19) |
|---|---|---|---|
| F | -18 600 | 16.1 | 4.8 |
| G | -12 000 | 49.1 | 10.4 |
| C | -6 700 | 43.8 | 3.6 |

- Site $F$ certainly does not outrank site $G$ as $F$ is less performing than $G$ on all three objectives (unanimous discordance = Pareto dominance).
- Site $F$ does not outrank site $C$ as $F$ is less performing than $C$ on objective "*Costs*" (-18 600 against -6 700) and objective "*Turnover*" (16.1 against 43.8).





# Indeterminate outranking situation

## Example

| Site | Costs (in €) | Turnover (0-81) | Work Cond. (0-19) |
|---|---|---|---|
| F | -18 600 | 16.1 | 4.8 |
| E | -34.800 | 49.1 | 15.7 |

- As site $F$ is less expensive than site $E$ (-18 600 against -34 800), but also, at the same time less advantageous on objective "*Turnover*" (16.1 against 49.1) and objective "*Work Cond.*" (4.8 against 15.7), one can neither confirm, nor reject this outranking situation.

  *This indeterminate situation is similar to a voting result where the number of votes in favour perfectly balance the number of votes in disfavour.*





# Indeterminate outranking situation

## Example

| Site | Costs (in €) | Turnover (0-81) | Work Cond. (0-19) |
|---|---|---|---|
| B | -17 800 | 29.5 | 9.9 |
| A | -35 000 | 70.6 | 10.2 |

- Same indeterminate situation is observed when comparing sites $B$ and $A$. On the one hand, $B$ is less expensive than site $A$ (-17 800 against -35 000), but, on the other hand, $B$ is less advantageous both on objective "*Turnover*" (29.5 against 70.6) ans on objective "*Work Cond.*" (9.9 against 10.2).
- Yet, are the grades 9.9 and 10.2 on the "*Work. Cond*" really different ?.





# Taking into account the performances' imprecision

## Definition (Discrimination thresholds)

The concept of discrimination threshold allows to take into account on each criterion (or objective) the:

- imprecision of our knowledge about present or past facts,
- uncertainty which necessarily affects our knowledge of the future,
- difficulties to quantify qualitative consequences.

## Taking into account the performances' imprecision

### Definition (Discrimination thresholds – continue)

- Performance discrimination thresholds allow us to model the fact that the numerical difference observed between the performances of two potential decision alternatives on a criterion (or objective) may be:
  - compatible with them being indifferent (indifference threshold)
  - warranting a clear preference of one over the other (preference threshold)
  - indicating a potential preference of one over the other (weak preference threshold),



## Best office site for the SME

- Let us reconsider the performance table of our best office choice problem:

| Site | Costs | Turnover | Work Cond. |
|---|---|---|---|
| B | -17 800 € | 29.5 | 9.9 |
| A | -35 000 € | 70.6 | 10.2 |

- A difference of 0.5 points on objective "*Work Cond.*" is still considered to compatible with an indifference judgment of the potential office sites,
- Hence, site *B* outranks site *A*, as the former is clearly less expensive (-17 800 against -35 000) and also more or less at least as good as *A* on objective "*Work Cond.*" (9.9 against 10.2, difference smaller than the supposed indifference threshold).



## Taking into account large performance differences

### Definition (Veto situations)

- The concept of veto situation allows us to take into account on each criterion (or objective):

  the presence of a negative performance difference large enough, to render insignificant the otherwise observed weighted majority of concordance of a preferential judgment.
- or, similarly:

  the presence of a positive performance difference large enough, to render insignificant the otherwise warranted weighted minority of concordance of a preferential judgment.





## Taking into account large performance differences

### Definition (Veto thresholds)

The concept of veto threshold allows us to model the fact that the performance difference observed between two potential decision alternatives on a criterion (or objective) may be:

either, attesting the presence of a counter-performance large enough to put to doubt a significantly affirmed outranking situation;

or, attesting the presence of an out-performance large enough to put to doubt a significantly refuted outranking situation.

## Revisiting the best office site problem

- Consider the performances of alternatives $A$ and $F$ with respect to the three objectives:

| Site | Costs | Turnover | Work. Cond. |
|---|---|---|---|
| A | -35 000 € | 70.6 | 10.2 |
| F | -18 600 € | 16.1 | 4.8 |

The outranking situation between $A$ and $F$ is indeterminate.

- The CEO of the SME considers that a performance difference of 50 points on the "Turnover" objective attests a veto situation.

Hence, the out-performance on objective "*Turnover*" of site $A$ over site $F$ ($70.6 - 16.1 = 54.6 > 50.0$ pts) resolves this indterminateness in favour of site $A$.

Similarly, site $F$ does certainly not outrank site $A$, as the counter-performance on objective "*Turnover*" is so high that it renders insignificant the fact that $F$ is less expensive ($-18600$ against $-35000$).



## Notation

- Let $X$ be a finite set of $p$ decision alternatives.
- Let $N$ be a finite set of $n > 1$ criteria supporting an increasing real performance scale from 0 to $M_i$.
- Let $0 \leqslant q_i < p_i < v_i \leqslant M_i + \epsilon$ represent resp. the indifference, the preference, and the veto discrimination threshold observed on criterion $i$.
- Let $w_i$ be the significance of criterion $i$.
- Let $W$ be the sum of all criterion significances.
- Let $x$ and $y$ be two alternatives in $X$.
- Let $x_i$ be the performance of $x$ on criterion $i$





## Performing marginally *at least as good as*

Each criterion $i$ is characterising a double threshold order $\geqslant_i$ on $A$ in the following way:

$$r(x \geqslant_i y) = \begin{cases} +1 & \text{if} \quad x_i + q_i \geqslant y_i \\ -1 & \text{if} \quad x_i + p_i \leqslant y_i \\ 0 & \text{otherwise.} \end{cases} \qquad (1)$$

$+1$ signifies that "$x$ is *performing at least as good as* $y$" on criterion $i$,

$-1$ signifies that "$x$ is *not performing at least as good as* $y$" on criterion $i$.

$0$ signifies that "it is *unclear* whether, on criterion $i$, $x$ is performing at least as good as $y$".





## Performing globally *at least as good as*

Each criterion $i$ contributes the significance $w_i$ of his "*at least as good as*" characterisation $r(\geqslant_i)$ to the characterisation of a global "*at least as good as*" relation $r(\geqslant)$ in the following way:

$$\boxed{r(x \geqslant y) \; = \; \sum_{i \in F} \left[ \frac{w_i}{W} \cdot r(x \geqslant_i y) \right]} \qquad (2)$$

$1.0 \geqslant r(x \geqslant y) > 0.0$ signifies $x$ is *globally performing at least as good as* $y$,

$-1.0 \leqslant r(x \geqslant y) < 0.0$ signifies that $x$ is *not globally performing at least as good as* $y$,

$r(x \geqslant y) = 0.0$ signifies that it is *unclear* whether $x$ is globally performing at least as good as $y$.

## Performing marginally and globally *less than*

Each criterion $i$ is characterising a double threshold order $<_i$ (*less than*) on $A$ in the following way:

$$r(x <_i y) \quad = \quad \begin{cases} +1 & \text{if} \quad x_i + p_i \leqslant y_i \\ -1 & \text{if} \quad x_i + q_i \geqslant y_i \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

And, the *global less than* relation ($<$) is defined as follows:

$$\boxed{r(x < y) = \sum_{i \in F} \left[ \frac{w_i}{W} \cdot r(x <_i y) \right]} \tag{4}$$

**Proposition**

*The global "less than" relation $<$ is the dual ($\not\geqslant$) of the global "at least as good as" relation $\geqslant$.*





## Marginal *considerably better* or *worse performing* situations

We define a single threshold order, denoted $\lll_i$ which represents *considerably less performing* situations as follows:

$$r(x \lll_i y) = \begin{cases} +1 & \text{if} \quad x_i + v_i \leqslant y_i \\ -1 & \text{if} \quad x_i - v_i \geqslant y_i \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

And a corresponding dual *considerablyly better performing* situation $\ggg_i$ characterised as:

$$r(x \ggg_i y) = \begin{cases} +1 & \text{if} \quad x_i - v_i \geqslant y_i \\ -1 & \text{if} \quad x_i + v_i \leqslant y_i \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$





## Global *considerably better* or *considerably worse performing* situations

A global *veto*, or *counter-veto* situation is defined as follows:

$$r(x \lll y) \quad = \quad \circledvee_{i \in F} r(x \lll_i y) \tag{7}$$

$$r(x \ggg y) \quad = \quad \circledvee_{i \in F} r(x \ggg_i y) \tag{8}$$

where $\circledvee$ represents the epistemic polarising (Bisdorff 1997) or symmetric maximum (Grabisch et al. 2009) operator:

$$r \circledvee r' = \begin{cases} \max(r, r') & \text{if} \quad r \geqslant 0 \wedge r' \geqslant 0, \\ \min(r, r') & \text{if} \quad r \leqslant 0 \wedge r' \leqslant 0, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$





## Characterising veto and counter-veto situations

1. $r(x \lll y) = 1$ iff there exists a criterion $i$ such that $r(x \lll_i y) = 1$ and there does not exist otherwise any criteria $j$ such that $r(x \ggg_j y) = 1$.
2. Conversely, $r(x \ggg y) = 1$ iff there exists a criterion $i$ such that $r(x \ggg_i y) = 1$ and there does not exist otherwise any criteria $j$ such that $r(x \lll_j y) = 1$.
3. $r(x \ggg y) = 0$ if either we observe no very large performance differences or we observe at the same time, both a very large positive and a very large negative performance difference.

**Lemma**

*$r(\not\lll)^{-1}$ is identical to $r(\ggg)$.*

## The bipolar outranking relation $\succsim$

From an epistemic point of view, we say that:

1. alternative $x$ *outranks* alternative $y$, denoted $(x \succsim y)$, if
   1.1 a weighted majority of criteria validates a global outranking situation between $x$ and $y$, and
   1.2 no considerable counter-performance is observed on a discordant criterion,
2. alternative $x$ *does not outrank* alternative $y$, denoted $(x \not\succsim y)$, if
   2.1 a weighted majority of criteria invalidates a global outranking situation between $x$ and $y$, and
   2.2 no considerably better performing situation is observed on a concordant criterion.





## Polarising the global "*at least as good as*" characteristic

The bipolar-valued characteristic $r(\succsim)$ is defined as follows:

$$r(x \succsim y) = r(x \geqslant y) \circledcirc r(x \not\lll_1 y) \circledcirc \ldots \circledcirc r(x \not\lll_n y)$$

Properties:

1. $r(x \succsim y) = r(x \geqslant y)$ if no very considerable positive or negative performance differences between $x$ and $y$ are observed,
2. $r(x \succsim y) = 1.0$ if $r(x \geqslant y) \geqslant 0$ and $r(x \ggg y) = 1.0$,
3. $r(x \succsim y) = -1.0$ if $r(x \geqslant y) \leqslant 0$ and $r(x \lll y) = 1.0$,



## Coherence of the bipolar-valued outranking concept

### Proposition

*The dual* $(\not\succsim)$ *of the bipolar outranking relation* $\succsim$ *is identical to the strict converse outranking* $\precnsim$ *relation.*

Proof: We only have to check the case where $r(x \lll_i y) \neq 0.0$ for all $i \in F$. If $r(x \lll y) \neq 0.0$:

$$\begin{aligned} r(x \not\succsim y) &= -r(x \succsim y) \;=\; -\big[r(x \geqslant y) \circledcirc -r(x \lll y)\big] \\ &= \big[-r(x \geqslant y) \circledcirc r(x \lll y)\big] \\ &= \big[r(x \not\geqslant y) \circledcirc -r(x \ggg y)\big] \\ &= \big[r(x < y) \circledcirc r(x \not\ggg y)\big] \;=\; r(x \precnsim y). \end{aligned}$$

Else, there exist conjointly two criteria $i$ and $j$ such that $r(x \lll_i y) = 1.0$ and $r(x \ggg_i y) = 1.0$ such that $r(x \succsim y) = r(x \not\succsim y) = r(x \precnsim y) = 0.0$. □

## Semantics of the bipolar valuation

The valuation $r(\succsim)$ has following interpretation:

- $r(\succsim(x, y)) = +1.0$ signifies that the statement $x \succsim y$ is certainly valid.
- $r(\succsim(x, y)) = -1.0$ signifies that the statement $x \succsim y$ is certainely not valid.
- $r(\succsim(x, y)) > 0$ signifies that the statement $x \succsim y$ is more valid than not valid.
- $r(\succsim(x, y)) < 0$ signifies that $x \succsim y$ is more not-valid than valid.
- $r(\succsim(x, y)) = 0$ signifies that the statement $x \succsim y$ is indeterminate.

## The bipolar outranking (Condorcet) digraph

### Definition

- We denote $\widetilde{G}(X, r(\succsim))$ the bipolar-valued digraph modelled by $r(\succsim)$ on the set of potential decision alternatives $X$.
- We denote $G(X, \succsim)$, the crisp digraph associated with $\widetilde{G}$ where we retain all arcs such that $r(x \succsim y) > 0$.
- $G(X, \succsim)$ is called the Condorcet or median cut digraph associated with $\widetilde{G}(X, r(\succsim))$.



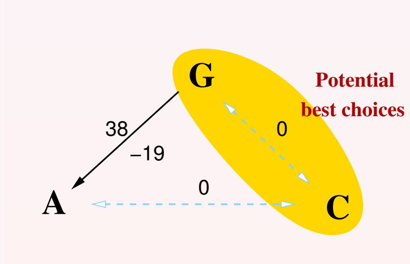
## The office site choice problem revisited

If we consider:

1. a preference threshold of 5 pts on objective "*Turnover*",
2. an indifference and a preference threshold 0.1 pt (resp. 0.5 pt) on objective "*Work. Cond.*",
3. and no veto situations,

the global characteristic (multiplied by 200) of the bipolar outranking relation $\succsim$ becomes:

| $r(\succsim)$ | A | C | G |
|---|---|---|---|
| A | 200 | 0 | -19 |
| C | 0 | 200 | 0 |
| G | 38 | 0 | 200 |








## The office site choice problem – continue

### Comment

- *The bipolar outranking characteristics show that:*
  1. *Site G is significantly at least as well performing as site A ($r(G \succsim A) = 38$)*
  2. *A is not significantly performing as well as site G ($r(A \succsim G) = -19$),*
  3. *No significant outranking situations may be confirmed, neither between sites G and C nor, between sites A and C.*
- *Hence G and C may be recommended as potential best choices.*

| *Site* | *Costs (in €)* | *Turnover (0-81)* | *Work Cond. (0-19)* |
|---|---|---|---|
| *C* | *-6 700* | *43.8* | *3.6* |
| *G* | *-12 000* | *49.1* | *10.4* |





## RUBIS : a best choice recommender system

- Traditionally, solving a best choice problem consists in finding the unique best decision alternative.
- In RUBIS , we adopt a modern recommender system's approach which shows a subset of alternatives which contains by construction the potential best alternative(s).
- If not reduced to a singleton, the actual "best choice", the recommendation has to be refined in a later decision process phase.

# Pragmatic principles for a best choice recommendation (BCR)

$\mathcal{P}_1$: Elimination for well motivated reasons.
Each eliminated alternative has to be outranked by at least one alternive in the BCR.

$\mathcal{P}_2$: Minimal size.
The BCR must be as limited in cardinality as possible.

$\mathcal{P}_3$: Efficient and informative.
The BCR must not contain a self-contained sub-recommendation.

$\mathcal{P}_4$: Effectively better.
The BCR must not be ambiguous in the sense that it is both a best choice as well as a worst choice recommendation.

$\mathcal{P}_5$: Maximally determined.
The BCR is, of all potential best choice recommendation, the most determined one in the sense of the characteristics of the bipolar outranking relation $\succsim$.



# Qualification of a BCR in $\widetilde{G}(X, r(\succsim))$

Let $Y$ be a non empty subset of $X$, called a choice in $\widetilde{G}$.

- $Y$ is called outranking (resp. outranked) iff for all non retained alternative $x$ there exists an alternative $y$ retained such that $r(y \succsim x) > 0.0$ (resp. $r(x \succsim y) > 0.0$).
- $Y$ is called independent iff for all $x \neq y$ in $Y$, we observe $r(x \succsim y) \leqslant 0.0$.
- $Y$ is an outranking kernel (resp. outranked kernel) iff $Y$ is an outranking (resp. outranked) and independent choice.
- $Y$ is an outranking (resp. outranked) hyper-kernel iff $Y$ is an outranking (resp. choice) containing chordless circuits of odd order $p \geqslant 1$.



# Translating the pragmatic principles in terme of choice qualification

$\mathcal{P}_1$: Elimination for well motivated reasons.
The BCR is an outranking choice.

$\mathcal{P}_{2+3}$: Minimal and stable recommendation.
The BCR is a hyper-kernel.

$\mathcal{P}_4$: Effectivity.
The BCR is a choice which is strictly more outranking than outranked.

$\mathcal{P}_5$: Maximal determination.
The BCR is the most determined one in the set of potential outranking hyper-kernels observed in a given bipolar outraking digraph $\widetilde{G}(X, r(\succsim))$.

**Theorem**

*Every bipolar strict outranking digraph $\widetilde{G}(X, r(\precsim\!\!\!\!/))$ admits at least one outranking and outranked hyper-kernel.*



# The Rubis best choice recommendation (RBCR)

- An strictly outranking hyper-kernel of maximal determination, if it exists, renders a RBCR.
- A RBCR verifies the five pragmatic principles.
- A RBCR is a recommended subset of alternatives which contains the best alternative, provided that it exists.
- A RBCR must not be confused with the actual best choice retained by the decision maker.
- Being only a best choice recommendation, the Rubis decision aid approach is only convenient in a progressive decision process.

## The complete, non-aggregated performance table

| Criterion | $w_i$ | $A$ | $B$ | $C$ | $D$ | $E$ | $F$ | $G$ |
|---|---|---|---|---|---|---|---|---|
| | | | | | Alternatives | | | |
| Costs | 45 | -35000 | -17800 | -6700 | -14100 | -34800 | -18600 | -12000 |
| Proximity | 32 | 100 | 20 | 80 | 70 | 40 | 0 | 60 |
| Visibility | 26 | 60 | 80 | 70 | 50 | 60 | 0 | 100 |
| Standing | 23 | 100 | 10 | 0 | 30 | 90 | 70 | 20 |
| Work. Space | 10 | 75 | 30 | 0 | 55 | 100 | 0 | 50 |
| Comfort | 6 | 0 | 100 | 10 | 30 | 60 | 80 | 50 |
| Parking | 3 | 90 | 30 | 100 | 90 | 70 | 0 | 80 |
| $W$ | 145 | | | | | | | |





## Performance discrimination thresholds

| Criterion | Thresholds (in points or €) indiff. | pref. | veto |
|---|---|---|---|
| Costs | 1000 € | 2500 € | 35 000 € |
| Proximity | 10 pts | 20 | 80 |
| Visibility | 10 | 20 | 80 |
| Standing | 10 | 20 | 80 |
| Work. Space | 10 | 20 | 80 |
| Comfort | 10 | 20 | 80 |
| Parking | 10 | 20 | 80 |





## The bipolar outranking digraph

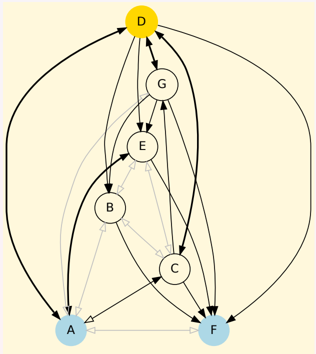
*Characteristics multiplied by* $W = 145$.

| $r(\succsim)$ | 'A' | 'B' | 'C' | 'D' | 'E' | 'F' | 'G' |
|---|---|---|---|---|---|---|---|
| 'A' | 145 | 0 | 55 | 43 | 113 | 0 | 0 |
| 'B' | 0 | 145 | 0 | -81 | 0 | 99 | -87 |
| 'C' | 0 | 0 | 145 | 67 | 0 | 87 | 15 |
| 'D' | 15 | 81 | 3 | 145 | 67 | 87 | 36 |
| 'E' | 75 | 0 | 0 | -15 | 145 | 43 | -61 |
| 'F' | 0 | -9 | -67 | -87 | -43 | 145 | -87 |
| 'G' | 0 | 133 | -15 | 145 | 79 | 87 | 145 |





## The RUBIS best choice recommendation




*Rubis Python Server (graphviz), R. Bisdorff, 2008*

| Choice | Determ. (%) | Qualification as $\succsim$ | $\precsim$ | indep. |
|---|---|---|---|---|
| $\{D\}$ | 51.0 | 3 | -87 | 145 |
| $\{A, G\}$ | 50.0 | 55 | 0 | 0 |
| $\{C, B, E\}$ | 50.0 | 15 | -9 | 0 |
| $\{A, F\}$ | 50.0 | 0 | 75 | 0 |

## The RUBIS best choice recommendation – continue

### Comment

- *The* RUBIS *best choice recommendation gives alternative* $\{D\}$*, a Condorcet winner, with a determination of* 51% *of the total significance of the criteria.*
- *A second and third potential BCR, a bit less determined, are given equivalently by the pair* $\{A, G\}$ *and the triplet* $\{C, B, E\}$.
- *A potential worst choice is given by the pair* $\{A, F\}$.





## Is alternative $G$ outranking alternative $A$?

| Criterion | $w_i$ | $G$ | $A$ | $G - A$ | sign. | veto |
|---|---|---|---|---|---|---|
| Costs | 45 | -12000 | -35000 | +23000 | +45 | -1 |
| Proximity | 32 | 60 | 100 | -40 | -32 | -1 |
| Visibility | 26 | 100 | 60 | +40 | +26 | -1 |
| Standing | 23 | 20 | 100 | -80 | -23 | +1 |
| Work Space | 10 | 50 | 75 | -25 | -10 | -1 |
| Comfort | 6 | 50 | 0 | +50 | +6 | -1 |
| Parking | 3 | 80 | 90 | -10 | +3 | -1 |
| $W$ | 145 | | | $r(G \succsim A) =$ | 0 | |





## Is alternative $C$ outranking alternative $A$?

| Criterion | $w_i$ | $C$ | $A$ | $C - A$ | sign. | veto |
|---|---|---|---|---|---|---|
| Costs | 45 | -6700 | -35000 | +28300 | +45 | -1 |
| Proximity | 32 | 80 | 100 | -20 | -32 | -1 |
| Visibility | 26 | 70 | 60 | +10 | +26 | -1 |
| Standing | 23 | 0 | 100 | -100 | -23 | +1 |
| Work Space | 10 | 0 | 75 | -75 | -10 | -1 |
| Comfort | 6 | 10 | 0 | +10 | +6 | -1 |
| Parking | 3 | 100 | 90 | +10 | +3 | -1 |
| $W$ | 145 | | | $r(C \succsim G) =$ | 0 | |





## Is alternative $D$ a significant Condorcet winner ?

### Exercise(s)

- *Alternative* $D$ *is outranking all the other office site alternatives.*

1. *Analyse in detail the outranking situation between alternatives* $D$ *and* $C$.
2. *What happens to the previous outranking situation, if a performance difference of 10 pts on the benefits criteria may not be anymore disregarded ?*
3. *Under what hypothesis may alternative* $C$ *become a better alternative than* $D$ *?*

# Conclusions

- Similarly to the MAVT, the outranking approach stresses the necessicity to follow a consistent and systematic approach for evaluating the performances of the potential decision alternatives.
- Similarly to the MAVT, the outranking approach allows to model costs and benefits with the help of multiple qualitative and/or quantitative performance criteria.
- Contrary to the MAVT, the outranking approach does not make the assumption that the evaluations on all the criteria must be commensurable in order to model global preferences.
- Contrary to the weighted scoring approaches, the significance of the criteria in the global outranking does not need to take into the type and scope of the marginal performance measurement scales.



Introduction Outranking Theory Recommender System Conclusions

# Conclusions – continue

- By adopting a pairwise comparison approach à la Condorcet, we abandon the idea of complete comparability and transitivity of the preferences and receive in return the independence of all preferential statements from irrelevant alternatives (see Arrows impossibility theorem in Lecture 2).
- Taking into account performance discrimination thresholds allows to efficiently model imprecision, uncertainties and even very large positive and negative differences in the performance data.
- The bipolar characteristic valuation in $[-1.0; +1.0]$ allows with the median value 0.0 to handle safely highly contradictory as well as missing data.
- RUBIS best choice recommendations like all modern recommender systems give a practical decision aid tool which avoids to force the hand of the decision maker with a definite unique normative result.