Best multiple criteria compromise choice: the Rubis outranking approach

MICS: AlgorithmicDecision Theory

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Introduction

The outranking situation

Definition

- We say that "a decision alternative *a* outranks a decision alternative *b*" if and only:
  1. There is a significant majority of criteria (or objectives) who warrant that *a* is perceived at least as good as *b* and,
  2. No considerable negative performance difference is observed between *a* and *b* on any criterion (or objective).
- We say that "a decision alternative *a* does not outrank a decision alternative *b*" if and only if:
  3. There is only a significant minority of criteria (or objectives) who warrant that *a* is perceived at least as good as *b* and,
  4. No considerable positive performance difference between *a* and *b* is observed on any criterion (or objective).
- Cases (2), respectively (4), are called veto, respectively counter–veto situations.

Best office choice

- Let us reconsider the best office choice problem from lecture 5.
- Below the performances of the seven potential office sites with respect to the three objectives:

Site	Costs (in €)	Turnover (0-81%)	Work Cond. (0-19%)
A	-35 000	70.6	10.2
B	-17 800	29.5	9.9
C	-6 700	43.8	3.6
D	-14 100	42.3	10.0
E	-34.800	49.1	15.7
F	-18 600	16.1	4.8
G	-12 000	49.1	10.4

Significant preferential judgment

Example

- The CEO of the SME judges the "*Costs*" and the cumulated "*Benefits*" objectives ("*Turnover*" and "*Working Conditions*") to be equi-significant for selecting the best office site.
- Hence, he considers that a concordant preferential judgment with respect to "*Costs*" and one of the two "*Benefits*" objectives is significant for him.

Certainly confirmed outranking situation

Example

Site	Costs (in €)	Turnover (0-81)	Work Cond. (0-19)
G	-12 000	49.1	10.4
F	-18 600	16.1	4.8

- Site G certainly outranks site F as G is at least as well performing than F on all three objectives (unanimous concordance = Pareto dominance).

Positively confirmed outranking situation

Example

Site	Costs (in €)	Turnover (0-81)	Work Cond. (0-19)
C	-6 700	43.8	3.6
B	-17 800	29.5	9.9

- Site C outranks site B as C is at least as well performing than B on objective "*Costs*" (-6 700 against -17 800) and on objective "*Turnover*" (43.8 against 29.5).

Confirmed outranking situation

Example

Site	Costs (in €)	Turnover (0-81)	Work Cond. (0-19)
G	-12 000	49.1	10.4
A	-35 000	70.6	10.2

- Site G outranks site A, as G is at least as well performing than A on objective "*Costs*" (-12 000 against -35 000) and objective "*Work Cond.*" (10.4 against 10.2).

Positively rejected outranking situation

Example

Site	Costs (in €)	Turnover (0-81)	Work Cond. (0-19)
F	-18 600	16.1	4.8
G	-12 000	49.1	10.4
C	-6 700	43.8	3.6

- Site F certainly does not outrank site G as F is less performing than G on all three objectives (unanimous discordance = Pareto dominance).
- Site F does not outrank site C as F is less performing than C on objective "*Costs*" (-18 600 against -6 700) and objective "*Turnover*" (16.1 against 43.8).

Indeterminate outranking situation

Example

Site	Costs (in €)	Turnover (0-81)	Work Cond. (0-19)
F	-18 600	16.1	4.8
E	-34.800	49.1	15.7

- As site F is less expensive than site E (-18 600 against -34 800), but also, at the same time less advantageous on objective "*Turnover*" (16.1 against 49.1) and objective "*Work Cond.*" (4.8 against 15.7), one can neither confirm, nor reject this outranking situation.

  This indeterminate situation is similar to a voting result where the number of votes in favour perfectly balance the number of votes in disfavour.

Indeterminate outranking situation

Example

Site	Costs (in €)	Turnover (0-81)	Work Cond. (0-19)
B	-17 800	29.5	9.9
A	-35 000	70.6	10.2

- Same indeterminate situation is observed when comparing sites B and A. On the one hand, B is less expensive than site A (-17 800 against -35 000), but, on the other hand, B is less advantageous both on objective "*Turnover*" (29.5 against 70.6) ans on objective "*Work Cond.*" (9.9 against 10.2).
- Yet, are the grades 9.9 and 10.2 on the "*Work. Cond*" really different ?

Taking into account the performances' imprecision

Definition (Discrimination thresholds)

The concept of discrimination threshold allows to take into account on each criterion (or objective) the:

- imprecision of our knowledge about present or past facts,
- uncertainty which necessarily affects our knowledge of the future,
- difficulties to quantify qualitative consequences.

Taking into account the performances' imprecision

Definition (Discrimination thresholds – continue)

- Performance discrimination thresholds allow us to model the fact that the numerical difference observed between the performances of two potential decision alternatives on a criterion (or objective) may be:
  - compatible with them being considered indifferent (indifference threshold)
  - warranting a clear preference of one over the other (preference threshold)

Best office site for the SME

- Let us reconsider the performance table of our best office choice problem:

Site	Costs	Turnover	Work Cond.
B	-17 800 €	29.5	9.9
A	-35 000 €	70.6	10.2

- A difference of 0.5 points on objective "*Work Cond.*" is still considered to compatible with an indifference judgment of the potential office sites,
- Hence, site B outranks site A, as the former is clearly less expensive (-17 800 against -35 000) and also more or less at least as good as A on objective "*Work Cond.*" (9.9 against 10.2, difference smaller than the supposed indifference threshold).

Taking into account large performance differences

Definition (Veto situations)

- The concept of veto situation allows us to take into account on each criterion (or objective):

  the presence of a negative performance difference large enough, to render insignificant the otherwise observed weighted majority of concordance of a preferential judgment.
- or, similarly:

  the presence of a positive performance difference large enough, to render insignificant the otherwise warranted weighted minority of concordance of a preferential judgment.

Taking into account large performance differences

Definition (Veto thresholds)

The concept of veto threshold allows us to model the fact that the performance difference observed between two potential decision alternatives on a criterion (or objective) may be:

either, attesting the presence of a counter-performance large enough to put to doubt a significantly affirmed outranking situation;

or, attesting the presence of an out-performance large enough to put to doubt a significantly refuted outranking situation.

Revisiting the best office site problem

- Consider the performances of alternatives A and F with respect to the three objectives:

Site	Costs	Turnover	Work. Cond.
A	-35 000 €	70.6	10.2
F	-18 600 €	16.1	4.8

  The outranking situation between A and F is indeterminate.

- The CEO of the SME considers that a performance difference of 50 points on the "Turnover" objective attests a veto situation.

  Hence, the out-performance on objective "*Turnover*" of site A over site F ($70.6 - 16.1 = 54.6 > 50.0$ pts) resolves this indterminateness in favour of site A.

  Similarly, site F does certainly not outrank site A, as the counter-performance on objective "*Turnover*" is so high that it renders insignificant the fact that F is less expensive (-18600 against -35000).

Notation

- Let X be a finite set of p decision alternatives.
- Let F be a finite set of n criteria supporting an increasing real performance scale from 0 to M_j ($j = 1, \ldots n$).
- Let $0 \leqslant ind_j < pr_j < v_j \leqslant M_j + \epsilon$ represent resp. the indifference, the preference, and the veto discrimination threshold observed on criterion j.
- Let w_j be the significance of criterion j.
- Let W be the sum of all criterion significances.
- Let x and y be two alternatives in X.
- Let x_j be the performance of x on criterion j

Performing marginally *at least as good as*

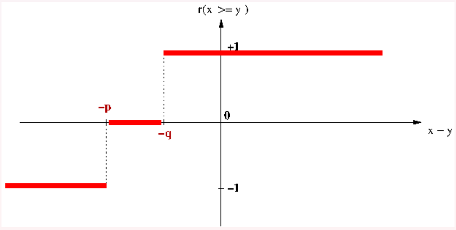
Each criterion j is characterizing a double threshold order $\geqslant_j$ on A in the following way:

$$r(x \geqslant_j y) = \begin{cases} +1 & \text{if} \quad x_j - y_j \geqslant -ind_j \\ -1 & \text{if} \quad x_j - y_j \leqslant -pr_j \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

$+1$ signifies x is *performing at least as good as* y on criterion j,

-1 signifies that x is *not performing at least as good as* y on criterion j.

0 signifies that it is *unclear* whether, on criterion j, x is performing at least as good as y.



Performing globally *at least as good as*

Each criterion j contributes the significance w_j of his "*at least as good as*" characterisation $r(\succsim_j)$ to the characterisation of a global "*at least as good as*" relation $r(\succsim)$ in the following way:

$$\boxed{r(x \succsim y) \;=\; \sum_{j \in F} \left[\frac{w_j}{W} \cdot r(x \succsim_j y) \right]} \qquad (2)$$

- $1.0 \geqslant r(x \succsim y) > 0.0$ signifies x is *globally performing at least as good as* y,
- $-1.0 \leqslant r(x \succsim y) < 0.0$ signifies that x is *not globally performing at least as good as* y,
- $r(x \succsim y) = 0.0$ signifies that it is *unclear* whether x is globally performing at least as good as y.

Performing marginally and globally *less than*

Each criterion j is characterising a double threshold order $<_j$ (*less than*) on A in the following way:

$$r(x \prec_j y) \quad = \quad \begin{cases} +1 & \text{if} \quad x_j + pr_j \leqslant y_i \\ -1 & \text{if} \quad x_j + ind_j \geqslant y_i \\ 0 & \text{otherwise.} \end{cases} \qquad (3)$$

And, the *global less than* relation $(\prec)$ is defined as follows:

$$\boxed{r(x \prec y) = \sum_{j \in F} \left[\frac{w_j}{W} \cdot r(x \prec_j y) \right]} \qquad (4)$$

Property

The global "less than" relation $\prec$ is the dual ($\not\succsim$) of the global "at least as good as" relation $\succsim$.

Marginal *considerably better* or *worse performing* situations

We define a single threshold order, denoted $\ll_j$ which represents *considerably less performing* situations as follows:

$$r(x \ll_j y) = \begin{cases} +1 & \text{if} \quad x_j + v_j \leqslant y_j \\ -1 & \text{if} \quad x_j - v_j \geqslant y_j \\ 0 & \text{otherwise.} \end{cases} \qquad (5)$$

And a corresponding dual *considerably better performing* situation $\gg_j$ characterised as:

$$r(x \gg_j y) = \begin{cases} +1 & \text{if} \quad x_j - v_j \geqslant y_i \\ -1 & \text{if} \quad x_j + v_j \leqslant y_i \\ 0 & \text{otherwise.} \end{cases} \qquad (6)$$

Global *considerably better* or *considerably worse performing* situations

A global *veto*, or *counter-veto* situation is defined as follows:

$$r(x \ll y) \quad = \quad \bigovee_{j \in F} r(x \ll_j y) \qquad (7)$$

$$r(x \gg y) \quad = \quad \bigovee_{j \in F} r(x \gg_j y) \qquad (8)$$

where $\ovee$ represents the epistemic polarising (Bisdorff 1997) or symmetric maximum (Grabisch et al. 2009) operator:

$$r \ovee r' = \begin{cases} \max(r, r') & \text{if} \quad r \geqslant 0 \wedge r' \geqslant 0, \\ \min(r, r') & \text{if} \quad r \leqslant 0 \wedge r' \leqslant 0, \\ 0 & \text{otherwise.} \end{cases} \qquad (9)$$

Characterising veto and counter-veto situations

1. $r(x \ll y) = 1$ iff there exists a criterion i such that $r(x \ll_i y) = 1$ and there does not exist otherwise any criteria j such that $r(x \gg_j y) = 1$.
2. Conversely, $r(x \gg y) = 1$ iff there exists a criterion i such that $r(x \gg_i y) = 1$ and there does not exist otherwise any criteria j such that $r(x \ll_j y) = 1$.
3. $r(x \gg y) = 0$ if either we observe no considerable performance differences or we observe at the same time, both a considerable positive and a considerable negative performance difference.

Lemma

$r(\not\ll)^{-1}$ is identical to $r(\gg)$.

The bipolar outranking relation $\succsim$

From an epistemic point of view, we say that:

1. alternative x *outranks* alternative y, denoted $(x \succsim y)$, if
   - 1.1 a significant majority of criteria validates a global outranking situation between x and y, and
   - 1.2 no considerable counter-performance is observed on a discordant criterion,
2. alternative x *does not outrank* alternative y, denoted $(x \not\succsim y)$, if
   - 2.1 a significant majority of criteria invalidates a global outranking situation between x and y, and
   - 2.2 no considerably better performing situation is observed on a concordant criterion.

Polarising the global "*at least as good as*" characteristic

The bipolar-valued characteristic $r(\succsim)$ is defined as follows:

$$\boxed{r(x \succsim y) = r(x \succeq y) \circledvee r(x \not\ll_1 y) \circledvee \ldots \circledvee r(x \not\ll_n y)}$$

Properties:

1. $r(x \succsim y) = r(x \succeq y)$ if no considerable positive or negative performance differences between x and y are observed,
2. $r(x \succsim y) = 1.0$ if $r(x \succeq y) \geqslant 0$ and $r(x \gg y) = 1.0$,
3. $r(x \succsim y) = -1.0$ if $r(x \succeq y) \leqslant 0$ and $r(x \ll y) = 1.0$,
4. The bipolar outranking relation $\succsim$ is weakly complete: either $r(x \succsim y) \geq 0$ or $r(y \succsim x) \geq 0$.

Coherence of the bipolar-valued outranking concept

Property

The dual $(\not\succsim)$ of the bipolar outranking relation $\succsim$ is identical to the strict converse outranking $\precnsim$ relation.

Proof: We only have to check the case where $r(x \ll_i y) \neq 0.0$ for all $i \in F$. If $r(x \ll y) \neq 0.0$:

$$\begin{aligned} r(x \not\succsim y) &= -r(x \succsim y) = -\big[r(x \succeq y) \circledvee -r(x \ll y)\big] \\ &= \big[-r(x \succeq y) \circledvee r(x \ll y)\big] \\ &= \big[r(x \not\succeq y) \circledvee -r(x \gg y)\big] \\ &= \big[r(x \prec y) \circledvee r(x \not\gg y)\big] = r(x \precnsim y). \end{aligned}$$

Else, there exist conjointly two criteria i and j such that $r(x \ll_i y) = 1.0$ and $r(x \gg_i y) = 1.0$ such that $r(x \succsim y) = r(x \not\succsim y) = r(x \precnsim y) = 0.0$. $\square$

Semantics of the bipolar valuation

The valuation $r(\succsim)$ has following interpretation:

- $r(x \succsim y) = +1.0$ signifies that the statement $x \succsim y$ is certainly valid.
- $r(x \succsim y) = -1.0$ signifies that the statement $x \succsim y$ is certainly invalid.
- $r(x \succsim y) > 0$ signifies that the statement $x \succsim y$ is more valid than invalid.
- $r(x \succsim y) < 0$ signifies that $x \succsim y$ is more invalid than valid.
- $r(x \succsim y) = 0$ signifies that the statement $x \succsim y$ is indeterminate.

The bipolar outranking (Condorcet) digraph

Definition

- We denote $\widetilde{G}(X, r(\succsim))$ the bipolar-valued digraph modelled by $r(\succsim)$ on the set of potential decision alternatives X.
- We denote $G(X, \succsim)$, the crisp digraph associated with $\widetilde{G}$ where we retain all arcs such that $r(x \succsim y) > 0$.
- $G(X, \succsim)$ is called the Condorcet or median cut digraph associated with $\widetilde{G}(X, r(\succsim))$.

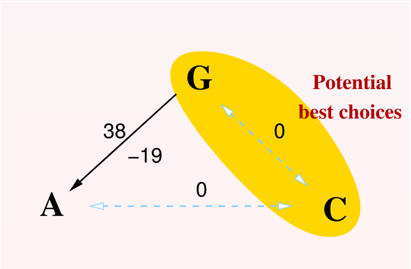
The office site choice problem revisited

If we consider:

1. a preference threshold of 5 pts on objective "*Turnover*",
2. an indifference and a preference threshold 0.1 pt (resp. 0.5 pt) on objective "*Work. Cond.*",
3. and no veto situations,

the global characteristic (multiplied by 200) of the bipolar outranking relation $\succsim$ becomes:

$r(\succsim)$	A	C	G
A	200	0	-19
C	0	200	0
G	38	0	200



The office site choice problem – continue

Comment

- *The bipolar outranking characteristics show that:*
  1. *Site G is significantly at least as well performing as site A* $(r(G \succsim A) = 38)$
  2. *A is not significantly performing as well as site G* $(r(A \succsim G) = -19)$,
  3. *No significant outranking situations may be confirmed, neither between sites G and C nor, between sites A and C.*
- *Hence G and C may be recommended as potential best choices.*

Site	*Costs (in €)*	*Turnover (0-81)*	*Work Cond. (0-19)*
C	*-6 700*	*43.8*	*3.6*
G	*-12 000*	*49.1*	*10.4*

1. Compare with potentially conflicting criteria
   - The outranking situation
   - Taking into account the performances' imprecision
   - Considering large performance differences
2. Theoretical foundation of the outranking approach
   - Overall preference concordance
   - Taking into account vetoes
   - The bipolar-valued outranking relation
3. The Rubis best-choice recommender system
   - Best-choice recommender system design
   - Resolving a best-choice problem
   - The RUBIS best-choice recommendation

Designing a best-choice recommender system

- Traditionally, solving a best-choice problem consists in finding the unique best decision alternative.
- In Rubis , we adopt a modern recommender system's approach which shows a subset of alternatives which contains by construction the potential best alternative(s).
- If not reduced to a singleton, the actual "best choice", the recommendation has to be refined in a later decision process phase.

Pragmatic principles for a best-choice recommendation (BCR)

$\mathcal{P}_1$: Elimination for well motivated reasons.
Each eliminated alternative has to be outranked by at least one alternive in the BCR.

$\mathcal{P}_2$: Minimal size.
The BCR must be as limited in cardinality as possible.

$\mathcal{P}_3$: Efficient and informative.
The BCR must not contain a self-contained sub-recommendation.

$\mathcal{P}_4$: Effectively better.
The BCR must not be ambiguous in the sense that it is both a best choice as well as a worst choice recommendation.

$\mathcal{P}_5$: Maximally determined.
The BCR is, of all potential best-choice recommendations, the most determined one in the sense of the characteristics of the bipolar-valued outranking relation $\succsim$.

Qualification of a BCR in an outranking digraph

Let Y be a non empty subset of X, called a choice in an outranking digraph $\widetilde{G}(X, r(\succsim)$.

- Y is called outranking (resp. outranked) iff for all non retained alternative x there exists an alternative y retained such that $r(y \succsim x) > 0.0$ (resp. $r(x \succsim y) > 0.0$).
- Y is called independent iff for all $x \neq y$ in Y, we observe $r(x \succsim y) \leqslant 0.0$.
- Y is an outranking kernel (resp. outranked kernel) iff Y is an outranking (resp. outranked) and independent choice.
- An outranking digraph $\widetilde{G}$ may not admit an outranking (resp. outranked) kernel if it contains a chordless odd circuit. The case given, we recursively break up all circuits at their weakest link.

Translating the pragmatic principles in terms of choice qualification

$\mathcal{P}_1$: Elimination for well motivated reasons.
The BCR is an outranking choice.

$\mathcal{P}_{2+3}$: Minimal and stable recommendation.
The BCR is a kernel.

$\mathcal{P}_4$: Effectivity.
The BCR is a choice which is strictly more outranking than outranked.

$\mathcal{P}_5$: Maximal determination.
The BCR is the most determined one in the set of potential outranking kernels observed in a given bipolar outranking digraph $\widetilde{G}(X, r(\succsim))$.

Property (BCR Decisiveness)

Every bipolar strict outranking digraph $\widetilde{G}(X, r(\not\precsim))$ without chordless odd circuit admits at least one outranking and one outranked kernel.

The Rubis best-choice recommendation (RBCR)

- A (strict) outranking kernel of maximal determination renders a RBCR. By default, we compute the RBCR on the strict (codual) outranking digraph where we previously break all chordless odd circuits.
- A RBCR verifies the five pragmatic principles.
- A RBCR is a recommended subset of alternatives which contains the best alternative, provided that it exists.
- A RBCR must not be confused with the actual best choice retained by the decision maker.
- Being only a best-choice recommendation, the Rubis decision aid approach is only convenient in a progressive decision process.
- Mind that enumerating and breaking chordless circuits, the case given, are operationally difficult problems. The same is true for enumerating (strict) outranking kernels. Hence, a RBCR may in general only be computed for decision problems involving less than 50 decision alternatives.

Introduction Outranking Theory Recommender System Conclusions

The performance table of the office choice problem

Criterion	w_i	A	B	C	D	E	F	G
		Alternatives						
Costs	45	-35000	-17800	-6700	-14100	-34800	-18600	-12000
Proximity	32	100	20	80	70	40	0	60
Visibility	26	60	80	70	50	60	0	100
Standing	23	100	10	0	30	90	70	20
Work. Space	10	75	30	0	55	100	0	50
Comfort	6	0	100	10	30	60	80	50
Parking	3	90	30	100	90	70	0	80
W	145							

Default performance discrimination thresholds

Criterion	Thresholds (in points or €) indiff.	pref.	veto
Costs	1000 €	2500 €	35 000 €
Proximity	10 pts	20	80
Visibility	10	20	80
Standing	10	20	80
Work. Space	10	20	80
Comfort	10	20	80
Parking	10	20	80

The bipolar outranking digraph

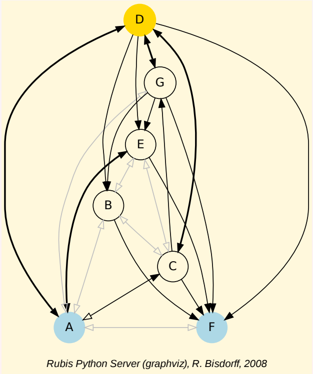
```
>>> from outrankingDigraphs import *
>>> t = PerformanceTableau('officeChoice')
>>> g = BipolarOutrankingDigraph(t)
>>> g.recodeValuation(-145,+145)
>>> g.showHTMLRelationTable(ndigits=0)
```

Characteristics multiplied by $W = 145$.

$r(\succsim)$	'A'	'B'	'C'	'D'	'E'	'F'	'G'
'A'	145	0	+145	+43	+113	0	0
'B'	0	145	0	-81	0	+145	-87
'C'	0	+0	145	+67	0	+145	+15
'D'	+15	+81	+3	145	+67	+145	+36
'E'	+75	0	0	-15	145	+145	-61
'F'	0	-145	-145	-145	-145	145	-145
'G'	0	+133	-15	+145	+79	+145	145

The RUBIS best-choice recommendation

```
>>> g.computeChordlessCircuits()
 [] # no chordless outranking circuits detected
>>> g.showBestChoiceRecommendation(CoDual=False)
>>> g.exportGraphViz(bestChoice=['D'],worstChoice=['A','F'])
```



Choice	Determ. (%)	Qualification as $\succsim$	$\not\succsim$	indep.
$\{D\}$	51.0	3	145	145
$\{A, G\}$	50.0	113	0	0
$\{C, B, E\}$	50.0	15	145	0
$\{A, F\}$	50.0	0	145	0

The RUBIS best-choice recommendation – continue

Comment

- *The outranking digraph here does not contain any chordless outranking circuit. Hence, we may compute the RBCR with a* `CoDual=False` *flag.*
- *The* RUBIS *best outranking choice recommends alternative* $\{D\}$, *a Condorcet winner, supported by* 51% *of the total significance of the criteria.*
- *A second and third potential BCR, but without a majority support, recommend the pair* $\{A, G\}$ *and the triplet* $\{C, B, E\}$.
- *A potential worst choice recommends the pair* $\{A, F\}$. *Alternative A (a weak Condorcet winner and looser) appears hence conjointly in a potential best, as well as worst, recommendation: a consequence of its weak comparability (high benefits combined with highest costs).*
- *Alternative G is nearly a weak Condorcet winner; only alternative C appears to be slightly better performing.*

Is alternative G outranking alternative A?

```
>>> g.showPairwiseComparison('G','A')
```

Criterion	w_i	G	A	$G - A$	sign.	veto
Costs	45	-12000	-35000	+23000	+45	-1
Proximity	32	60	100	-40	-32	-1
Visibility	26	100	60	+40	+26	-1
Standing	23	20	100	-80	-23	+1
Work Space	10	50	75	-25	-10	-1
Comfort	6	50	0	+50	+6	-1
Parking	3	80	90	-10	+3	-1
W	145			$r(G \succsim A) =$	0	

Is alternative C outranking alternative A?

```
>>> g.showPairwiseComparison('C','A')
```

Criterion	w_i	C	A	$C-A$	sign.	veto
Costs	45	-6700	-35000	+28300	+45	-1
Proximity	32	80	100	-20	-32	-1
Visibility	26	70	60	+10	+26	-1
Standing	23	0	100	-100	-23	+1
Work Space	10	0	75	-75	-10	-1
Comfort	6	10	0	+10	+6	-1
Parking	3	100	90	+10	+3	-1
W	145			$r(C \succsim G) =$	0	

Is alternative D a significant Condorcet winner ?

Exercise(s)

Alternative D is outranking all the other office site alternatives.

1. *Analyse in detail the outranking situation between alternatives D and C.*
2. *What happens to the previous outranking situation, if a performance difference of 10 pts on the benefits criteria may not be anymore disregarded ?*
3. *Under what hypothesis may alternative C become a better alternative than D ?*
4. *What becomes the BCR if the CEO would consider all his three decision objectives as equally important ?*

Conclusions

- Similarly to the MAVT, the outranking approach stresses the necessicity to follow a consistent and systematic approach for evaluating the performances of the potential decision alternatives.
- Similarly to the MAVT, the outranking approach allows to model costs and benefits with the help of multiple qualitative and/or quantitative performance criteria.
- Contrary to the MAVT, the outranking approach does not make the assumption that the evaluations on all the criteria must be commensurable in order to model global preferences.
- Contrary to the weighted scoring approaches, the significance of the criteria in the global outranking does not need to take into account the type and scope of the marginal performance measurement scales.

Conclusions – continue

- By adopting a pairwise comparison approach à la Condorcet, we abandon the idea of complete comparability and transitivity of the preferences and receive in return the independence of all preferential statements from irrelevant alternatives (see Arrows impossibility theorem in Lecture 2).
- Taking into account performance discrimination thresholds allows to efficiently model imprecision, uncertainties and even considerable positive and negative differences in the performance data.
- The bipolar characteristic valuation allows, with the median value 0.0, to handle safely highly contradictory, imprecise, as well as missing data.
- RUBIS best-choice recommendations, like all modern recommender systems, give a practical decision assistance which avoids forcing the hand of the decision maker with a definite unique normative choice.